

P18. This theorem says prime numbers must be of the form  $6n \pm 1$ . Unfortunately, there are also composite numbers of the same form. For example:

When  $n = 3$ ,  $6n \pm 1 = 19$  and  $17$ , both of which are prime numbers.

When  $n = 6$ ,  $6n \pm 1 = 37$  and  $35$ , only one of which is a prime number.

When  $n = 24$ ,  $6n \pm 1 = 145$  and  $143$ , both of which are composite numbers.

P19. The expression  $x^2 - x + 17$  will produce prime numbers for the positive integers  $x = 1, 2, 3, \dots, 15, 16$ .

The sixteen prime numbers produced are 17, 19, 23, 29, 37, 47, 59, 73, 89, 107, 127, 149, 173, 199, 227, and 257.

P20. a. The sixth term of the series, 333,337, is a prime number.

The seventh term of the series, 3,333,337, is a composite number.

$$3,333,337 = 7 \cdot 31 \cdot 15361$$

Therefore, only one additional prime number can be added to the series.

b. There are several possible answers. Here are two:

The first is a long sequence.

31 is a prime number

331 is a prime number

3331 is a prime number

33,331 is a prime number

333,331 is a prime number

3,333,331 is a prime number

33,333,331 is the composite number  $17 \cdot 19607843$

The second answer may seem sneaky, but it does satisfy the conditions.

37 is a prime number

337 is a prime number

3337 is a prime number

33,337 is a prime number

333,337 is a prime number

3,333,337 is the composite number  $7 \cdot 31 \cdot 15361$

P21. One could make a good argument for either of the two formulas being “best.”

The characteristics of Euler’s formula are straightforward. The formula produces forty unique prime numbers before producing a composite number.

The characteristics of Escott’s formula are less straightforward. The formula produces seventy-nine prime numbers before producing a composite number. However, thirty-nine of the seventy-nine prime numbers are produced twice. Thus Escott’s formula produces only forty unique prime numbers, the same as Euler’s formula.

Because most mathematicians find elegance in simplicity, they would likely prefer Euler’s formula.