

Marginal Notes

Goldbach's Conjecture

Suppose all your mathematics homework must be done without using the Web, a computer, a calculator, or a light bulb—candles are OK. Television and radio are out of the question. Suppose also that there are no cell phones or telephones. If you get stuck and need some help, face-to-face communication is the only possibility. The face you want to communicate with had better be within walking distance because there are no automobiles. If you have a really great idea, you can write a letter to a recognized expert and hope that you receive an answer and that the expert doesn't claim your idea was really his idea. It was under conditions like these in 1742 that the Prussian amateur mathematician, Christian Goldbach, wrote a letter to the brilliant Swiss mathematician, Leonhard Euler. His letter contained a phrase written in the margin that would haunt mathematicians for more than 260 years. Mathematicians occasionally do some of their best work in the margins. The phrase was:

"Es scheint wenigstens, daß eine jede Zahl, die größer ist als 2, ein aggregatum trium numerorum primorum sey."

Fortunately, Euler understood both German and Latin. He knew the phrase meant:

"It seems at least that every number that is greater than 2 is a sum of three prime numbers."

This phrase is known as **Goldbach's Conjecture**. At first glance, the conjecture seems to be true. For example:

$$\begin{aligned}12 &= 7 + 3 + 2 \\21 &= 11 + 7 + 3 \\22 &= 17 + 3 + 2 \\101 &= 59 + 37 + 5\end{aligned}$$

P1. a. Write six additional examples that suggest Goldbach's Conjecture is true.

b. Provide two examples that *prove* Goldbach's Conjecture is false.

How could Goldbach have missed such simple exceptions to his conjecture? Did he waste Euler's time and embarrass himself by writing the letter? Not at all—there is a good explanation. Euler, the father of thirteen children, is the most prolific mathematician of all time. He realized that Goldbach considered 1 a prime number. This was an easy mistake to make in the 1740s, when communication among mathematicians was so difficult. If 1 is a prime number, then the conjecture appears true just as Goldbach suggested. Euler didn't immediately realize that the simple-sounding conjecture was both