

Sequences and Numeric Black Holes

Integer Sequences

What are the next two letters of the sequence $o, t, t, f, f, s, s, \dots$? If you can determine the rule that defines the sequence, the answer is obvious.

A sequence is an ordered set of items. The items are called the **terms of the sequence**. The alphabet is a sequence. So are the positive integers and the multiples of 5. A sequence may be infinite like the positive integers or finite like the alphabet. An important fact about a sequence is that there is some rule being followed that determines each term of the sequence.

For example, the sequence 1, 3, 5, 7, 9, 11, 13, ... contains the odd integers. You can state the rule defining this sequence as “all of the odd integers in ascending order.” Another rule for defining the same sequence is “Let the first term be 1. Next add 2 to the first term to get the second term of the sequence. Continue to add 2 to each new term to get the next term.” The rule defining the sequence $o, t, t, f, f, s, s, \dots$ is “the first letters of the words spelling the numbers one through ten.” Did you correctly determine that the next two letters were e and n ?

A question commonly seen in mathematics and on aptitude and achievement tests is one that shows part of a sequence and asks you to determine the next terms of the sequence. For example:

What are the next two terms of the sequence 2, 3, 5, 7, 11, 13, 17, ... ?

Because the sequence is a list of the first seven prime numbers, a rule defining the sequence must be “all of the prime numbers.” Thus the next two terms are 19 and 23.

What are the next two terms of the sequence 1, 4, 9, 16, 25, 36, 49, 64, ... ?

Because the sequence is a list of the squares of the first eight positive integers, a rule defining the sequence must be “the squares of all the positive integers.” Thus the next two terms are 81 and 100.

If you can determine the rule that defines a sequence, adding additional terms is easy. How do you determine the rule? Sometimes, as in the previous two examples, you can do it “by inspection.” That means look at the sequence, make an intelligent guess at the rule, and then verify that it produces the terms that are in the sequence. If it works for all the terms you are given, then you can use it to extend the sequence.

P1. What are the next two terms in each of the following sequences? What is a rule that defines each sequence?

a. 1, 2, 4, 8, 16, 32, 64, ...

b. 3, 14, 25, 36, 47, 58, ...

c. 1, 8, 27, 64, 125, ...