

Creator Numbers and the 113 Mystery

Creator Numbers

How many positive integers can be expressed using only the integer 1, the operations $+$, $-$, \cdot , $/$, or exponentiation? For example:

$$11 = (1+1+1+1+1+1+1+1+1+1+1)$$

$$35 = (1+1+1+1+1)(1+1+1+1+1+1+1)$$

There's no doubt that all integers can be expressed like this. However, the mathematical term for doing so is *boring*. Let's make the task more interesting. Answer the same question, but also require that a minimum number of 1s be used in each expression. Then the previous examples become:

$$11 = (1+1+1)^{1+1} + 1+1 \quad 11 \text{ is expressed with seven 1s.}$$

$$35 = (1+1+1+1+1+1+1)^{1+1} - 1 \quad 35 \text{ is expressed with nine 1s.}$$

Try writing a few expressions on your own.

P1. Express each of the following numbers using only the integer 1, the operations $+$, $-$, \cdot , $/$, or exponentiation. Your expression should contain only the minimum number of 1s required to express the number.

- | | |
|-------|--------|
| a. 9 | d. 21 |
| b. 26 | e. 51 |
| c. 62 | f. 113 |

The creator number for an integer N is the least number of integers that can be used to express N according to a given set of conditions for the integers.

Look again at the previous examples.

The creator number of 11 is **7**, because $11 = (1+1+1)^{1+1} + 1+1$ requires **seven** 1s.

The creator number of 35 is **9**, because $35 = (1+1+1+1+1+1+1)^{1+1} - 1$ requires **nine** 1s.

Mathematicians often use symbols to represent an expression or definition. We will use \heartsuit to represent the creator number. Then we can write $\heartsuit(11) = 7$ and $\heartsuit(35) = 9$. If you aren't sure that nine is the least number of 1s needed to express 35, you can write $\heartsuit(35) \leq 9$. This indicates that the creator number is less than or equal to nine.

As long as the meaning of a symbol is clear, the choice of a symbol is arbitrary. The only important restriction is that the symbol doesn't already have another common mathematical meaning such as the symbols $+$, \neq , π , and \geq . Instead of using \heartsuit to represent the creator number, we might have chosen \odot . Then we would write $\odot(11) = 7$ and $\odot(35) = 9$. Let's stick with \heartsuit .