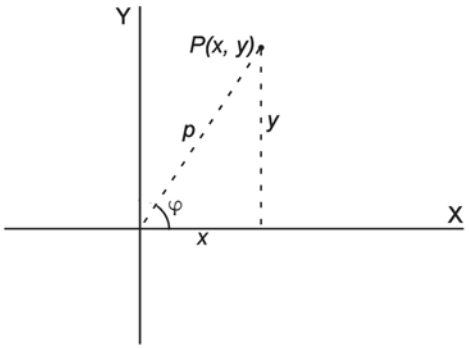
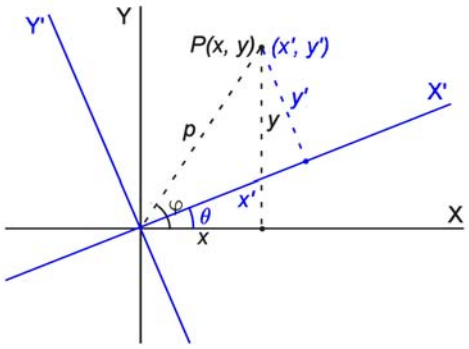
…Even Easier After Rotating the Axes

Here is a set of coordinate axes with point P having coordinates (x, y). If the distance from point P to the origin is p, we can write:

$$x = p \cos \varphi$$
$$y = p \sin \varphi$$





Here is the same set of coordinate axes rotated through angle θ. In a similar manner, we can write the coordinates of point P (which hasn't rotated) in the rotated coordinate system. Notice that the value of p has not changed.

$$x' = p \cos (\varphi - \theta)$$
$$y' = p \sin (\varphi - \theta)$$

Next use a trigonometric identity for the difference of two angles and substitution to produce:

$$\begin{aligned} x' = p \cos (\varphi - \theta) &= p (\cos \varphi \cos \theta + \sin \varphi \sin \theta) \\ &= p \cos \varphi \cos \theta + p \sin \varphi \sin \theta \\ &\mathbf{x' = x \cos \theta + y \sin \theta} \end{aligned}$$

$$\begin{aligned} y' = p \sin (\varphi - \theta) &= p (\sin \varphi \cos \theta - \cos \varphi \sin \theta) \\ &= p \sin \varphi \cos \theta - p \cos \varphi \sin \theta \\ &\mathbf{y' = y \cos \theta - x \sin \theta} \end{aligned}$$

Thus when the coordinates of P in the original coordinate system are (x, y), these equations produce the coordinates of P in the rotated coordinate system as (x', y').

Can you reverse the process? Sure, rather easily in fact. If the coordinates of P in the rotated coordinate system are (x', y'), you can solve the two previous equations to obtain the value of the coordinates using the original axes. You'll be asked to do this in Problem P22. The results will be:

$$\mathbf{x = x' \cos \theta - y' \sin \theta}$$
$$\mathbf{y = x' \sin \theta + y' \cos \theta}$$

What are the new values of A, B, C, D, E, and F in the new coordinate system? Finding them is straightforward, but it's also a bit tedious. To do this, substitute the above two