

Moving the Plane

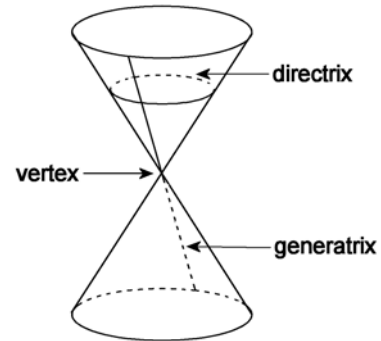
The examples in the previous section of this chapter are similar to problems in the first chapter of this book. That won't last. Next consider two other objects that have several possible intersections—a plane and a circular conical surface.

A **conical surface** is a surface which can be generated by a moving line which always passes through a fixed point and intersects a fixed curve. A **circular conical surface** is a conical surface whose fixed curve is a circle.

The fixed point is called the **vertex**.

The fixed curve is called the **directrix**.

The moving line which generates the surface is called the **generatrix**.



Now it's time to call upon your imagination. Imagine a circular conical surface and a plane moving freely in three-dimensional space. Can you see them? Can you see them tumbling, rolling, and floating—passing through each other as they intersect?

Did you notice that the two shapes always intersect? **There is no position of a plane and a circular conical surface in three-dimensional space in which the two surfaces do not intersect.** If you think you've found such a position, remember that both surfaces are infinite and can be extended. Eventually, they will intersect.

One goal of this chapter is to determine all of the different intersections that can be photographed with a stop-motion camera as the plane and circular conical surface intersect in space. Before reading further, take a few minutes to try to describe some of the intersections.

You probably realize that the intersection is always a two-dimensional figure. That's because one of the two surfaces is a plane, and the intersection of a plane with any other surface must lie entirely in the plane.

As with the previous problems in this chapter, one good way to approach this problem is to imagine the plane and the circular conical surface in specific, well-defined positions. Note that in the remainder of this chapter, the phrase “circular conical surface” is often abbreviated as “conical surface.” In this chapter, both phrases have the same meaning.