

Visualizing Algebra

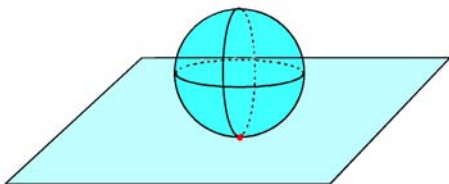
Moving through a Plane

This chapter is for those who have completed or are taking a second course in algebra. If you're not in this group, you will benefit most by completing a first course in algebra and then beginning this chapter when you begin a second course in algebra.

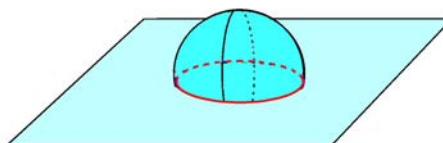
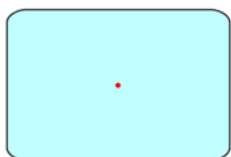
We're going to return to some of the activities we already thought about in the first chapter, but this time we'll stay in our familiar three-dimensional world and make use of a stop-motion digital camera. We're going to determine what shapes are made by the intersection of certain objects with a plane as they move through the plane. Photographing specific instances of the movement will give us a picture of the intersection at specific times.

For example, what happens when a sphere is dropped through a plane? Assume that the word "dropped" means that the object moves through the plane along an axis that is perpendicular to the plane.

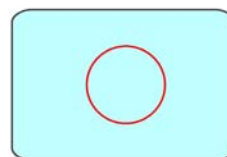
When a sphere is dropped through a plane there are two possible shapes for the intersection. Can you tell what they are? Assume stop-motion photographs of the intersections are taken in the following two instances. The points of intersection are shown in red. What will the photographs look like?



A photograph of this instance looks like a single point.



A photograph of this instance looks like a circle.



In fact, a sphere dropped through a plane always intersects the plane in either a circle or a point. Note that our rules in this chapter are different from those in the first chapter. In this chapter we are only interested in our stop-motion photographs. In the first chapter, the intersection of a sphere passing through a plane would be described as a point which expanded into a circle with a radius that increased to the radius of the sphere. The circle's