

Geometric Constructions

Euclid's *Elements*

This chapter is for those who have completed or are taking a course in geometry. If you're not in that group but you have a good imagination and are willing to experiment, then this chapter is for you as well. Creating geometric constructions could be classified as a one-person game. The process is fun and challenging, and success produces both personal satisfaction and interesting drawings.

Can you imagine writing a book that is so comprehensive and complete that it replaces all other books on the same topic? Librarians would keep the other books on their shelves because that's what librarians do, but no one would ever look at them. Everyone interested in or learning about the topic would use your book. Are there any books like this? The *Harry Potter* series is popular all over the world, but *Harry Potter* hasn't replaced all other literature or even all other children's books. The Bible is the primary book of the Christian religion, but it hasn't replaced all other religious books. There are many religions in the world and many of them have a different primary book. Some don't have any book at all.

In about 300 B.C. Euclid wrote a series of thirteen mathematics books called *Elements*—a single chain of 465 propositions comprising plane and solid geometry, number theory, and Greek geometric algebra. Euclid's work was so comprehensive that it replaced all known works of a similar nature. There was no need for any other text about mathematics—Euclid's *Elements* had it all and presented it clearly and concisely. Note that *Elements* is not only about geometry. It included all mathematics known at the time. A sketch of Euclid is on the cover of this book.

Many of the propositions in *Elements* involved geometric constructions. In this chapter, we will be working with the fourteen construction problems proposed in Book I of *Elements*.

The first fourteen ruler and compass construction problems in *Elements*, in the order presented by Euclid, are:

- C1. Construct an equilateral triangle having a given line segment as one side.
- C2. Construct a line segment congruent to a given a line segment and with a given point as one endpoint.
- C3. Given ray \overline{AB} and line segment \overline{CD} , construct point E on \overline{AB} such that $\overline{AE} \cong \overline{CD}$.
- C4. Construct the angle bisector of a given angle.
- C5. Construct the midpoint of a given line segment.